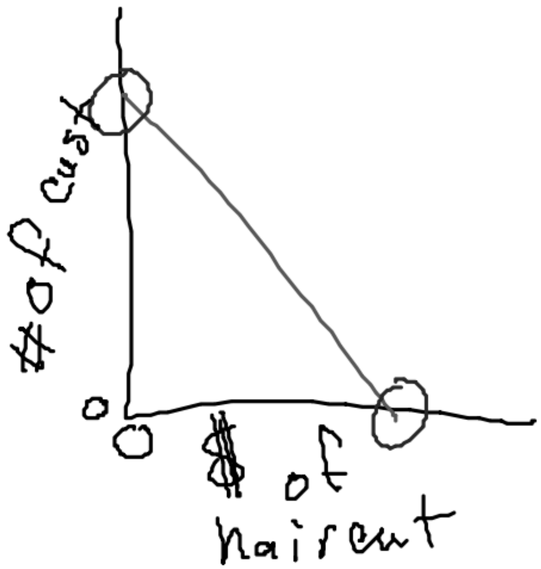
Chapter 4 Functions, Relations, and Transformations p. 171 11/7

4.1 Interpreting Graphs



y-intercept The largest # of cust. that could get haircuts

X-intercept The cost at which a haircut is ~~too~~ expensive for anyone.

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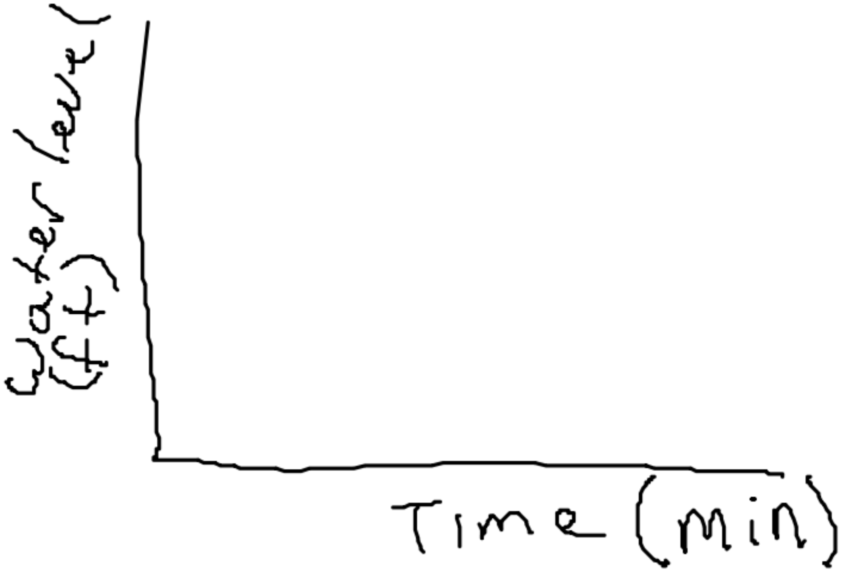


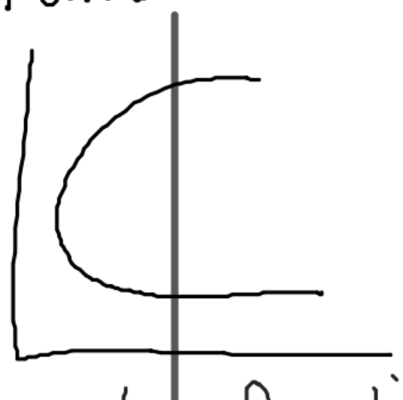
4.2 Function Notation p. 178 11/9

relation - any relationship between two variables

function - a type of relation that pairs <u>at</u> <u>most</u> one y with each x

Vertical line test - if a vertical line crosses a relation <u>at</u> <u>most</u> one time then it is a function

function



not a function

function notation $f(x)$

dependent variable

"f of x"

you have a function "f" which is dependent on x

$f(x)$ $g(x)$ $h(x)$

$x(x)$ $y(x)$ $z(x)$

NO

Ex $f(x) = \frac{2x+5}{x-3}$ 3, 6, 7, 9

$f(8) = \frac{2(8)+5}{(8)-3} = \frac{21}{5} = 4\frac{1}{5} = 4.2$

$f(-7) = \frac{2(-7)+5}{(-7)-3} = \frac{-9}{-10} = \frac{9}{10}$

$g(1) = 3$ y where $x=1$

$g(-2) = 0$ $g(4) = 3$

Inv a) Function

b) No

c) Yes♡

d) Yes

e) yeS

f) NO

g) NO

h) Yes

i) Yes

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$f(x) = 3x - 4$

$f(x) = 7 \quad x = ?$

$7 = 3x - 4$

$2f(22) = 2 \cdot f(22)$

Rene Descartes

18 5 14 5 4 5 19 3 1 18 20 5 19

Slope-intercept form $y = a + bx$

point-slope form $y = y_1 + b(x - x_1)$

slope $= b$

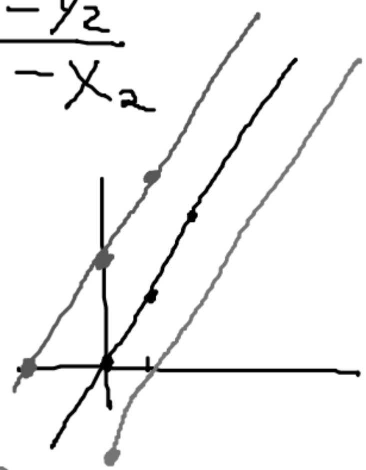
$$b = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_2}{x_1 - x_2}$$

Inv.

$y = 2x$

$y = 3 + 2x$

$y = -3 + 2x$



Moving a line up is the same as moving the line to the left.

Moving a line down is the same as moving the line to the right.

$y = 3 + 2x$

$\frac{y-3}{2} = \frac{2x}{2}$

$\frac{1}{2}y - \frac{3}{2} = x$

$\frac{1}{2}y + \frac{3}{2} = x$

Translations

Translation of a function

$y = f(x)$

$y = f(x-h) + k$

$y - k = f(x-h)$

h = horizontal movement

k = vertical movement

$y = f(x-3) + 6$

$\leftrightarrow$ right 3

up 6

$y = f(x-h) + k$ $\quad +k \quad$ $y = 2x - 3$ $\quad$ $\frac{2x-3}{2} = x - \frac{3}{2}$

$= \frac{y}{2}$

$k = -3$ down $\quad h \quad$ $y = 2\left(x - \frac{3}{2}\right)$

$h = \frac{3}{2}$ right $\quad$ $h = \frac{3}{2}$

$\frac{y}{2} = \frac{2x-3}{2}$

$y/4 = 2\left(x - (3/2)\right)$ $\quad$ $\cancel{2} \cdot \frac{y}{\cancel{2}} = \left(x - \frac{3}{2}\right) \cdot 2$

$y = 2\left(x - \frac{3}{2}\right)$

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$y = y_1 + b\,(x - x_1)$ point-slope form

$y = k + f(x - h)$ translation of a function vertically + horizontally

EX. Describe how the graph of $f(x) = 4 + 2(x - 3)$ is a translation of the graph of $f(x) = 2x$.

moves right 3

moves ~~down~~ up 4

$f(x) = 0 + 2(x - 0)$

$(0,0)$ $(3,4)$

(x,y) (h,k)

$y = 2(x-1)$

$y = 2x - 2$

$y = 4 + 2(x-3)$

$y = 4 + 2x - 6$

$y = -2 + 2x$

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$(x + h)$

4) $(-5.2, 3.18)$ $(1.4, -4.4)$

$$\frac{3.18 - {}^-4.4}{-5.2 - 1.4} = \frac{7.58}{-6.6} = -1.15$$

$y = 3.18 - 1.15(x + 5.2)$

b) $k = 2$

$y = 3.18 - 1.15(x + 5.2) + 2$

$y = 5.18 - 1.15(x + 5.2)$

5a) $y = 4.7x$ $\quad k = -3$ $\quad y = 4.7x - 3$

$h = 2$ $\quad y = -2.8(x - 2)$

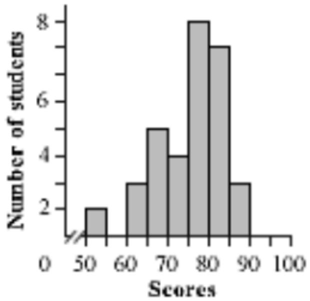
$k = 4$ $\quad h = -1.5$ $\quad y = -(x - {}^-1.5) + 4$

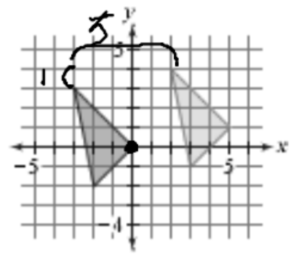
$(x + 1.5)$

3b) $f(x) = -2x$

$-3 + f(x-3) \qquad -3 + {}^{-}2(x-3) = -3 - 2x + 6 = -2x + 3$

$5 + f(x+1) \qquad 5 + {}^{-}2(x+1) = 5 - 2x - 2 = -2x + 3$

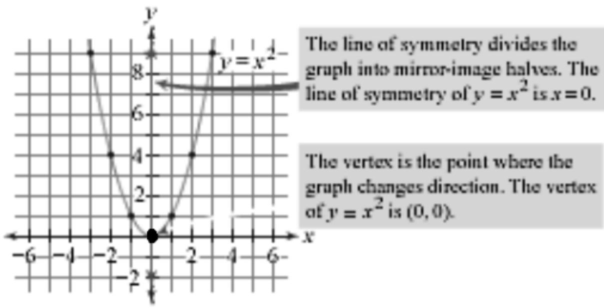




$h = 5$
$k = 1$

corner = Vertex (one)
vertices (multiple)

transformation – translations, reflection, rotation, stretch or shrink



quadratic = parabola

(0, 0)   (3, 4)

h = 3
k = 4

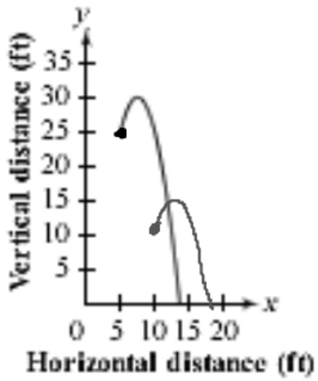
h goes w/ X

$y = x^2$ parent function

$y = (x-h)^2 + k$ $y = (x-3)^2 + 4$

11/22 symmetry = identical in parts

Ex.



- $f(x)$
- $f(x-5)-15$

$y=x^2$ U

$y=x^2$

$y=(x-h)^2+k$

$(y-k)=(x-h)^2$

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4.5 Reflections and the Square Root Family p. 201 11/28

reflection - mirror image

across the y-axis

across the x-axis

radicand

$y = \sqrt{x}$

$y_1 = $ 2nd x^2 X)

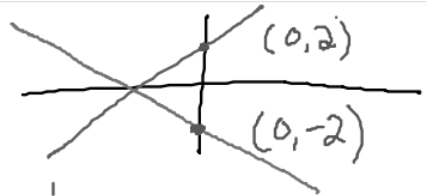
$y_1 = \sqrt{(x)}$

Friendly Window: **Xmin** -9.4, **Xmax** 9.4, **Xscl** 1, **Ymin** -6.2, **Ymax** 6.2, **Yscl** 1
After setting the window values, press ZOOM MEMORY 2:ZoomSto.
To use it again, press ZOOM MEMORY 3:ZoomRcl.

To get Y1, press Y-Vars 1:Function 1:Y1.

Inv. a) $y_1 = 0.5x + 2$

$y_2 = -y_1(x)$



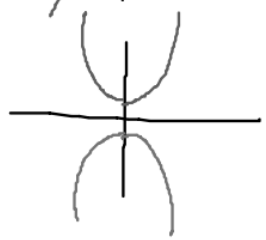
d) reflects across the X-axis

b) $y_1 = -2x - 4$

$y_2 = -y_1(x)$

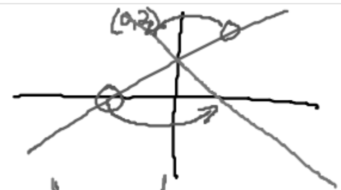
c) $y_1 = x^2 + 1$

$y_2 = -y_1(x)$



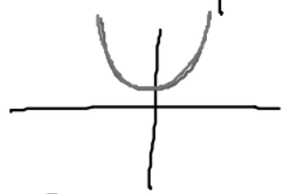
2 a) $y_1 = 0.5x + 2$

$y_2 = Y_1(-x)$



c) $Y_1 = x^2 + 1$

$y_2 = Y_1(-x)$



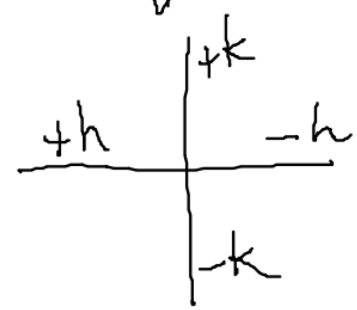
d) reflects across the y-axis

$y = -\sqrt{x-h} + k$

$y = \sqrt{x}$

$y - k = \sqrt{x-h}$

$y = \sqrt{x-h} + k$



Given $y = f(x)$

$y = -f(x)$ reflects across the x-axis

$y = f(-x)$ reflects across the y-axis

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4.6 Stretches, Shrinks, and the Absolute Value Family   p. 209   12/2

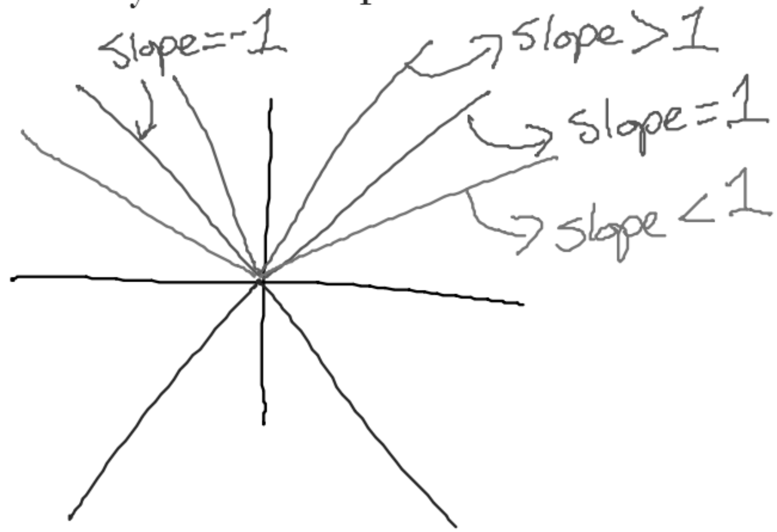
$|-7.4| = 7.4$

$y = |x|$ $y = |-12| = 12$

$y = |x-h| + k$

$y = -|x-h| + k$

y = |x|



vertical stretch = horizontal shrink

horizontal stretch = vertical shrink

Ex A a) $y = 2|x|$ $\qquad$ $y = \frac{1}{2}|x|$

$y=|x|$ $(1, 1)$ $(1, 2)$ $\qquad$ $(1,1)$ $(1, \frac{1}{2})$

$(-3, 3)$ $(-3, 6)$ $\qquad$ $(-4, 4)$ $(-4, 2)$

$y = 2f(x)$ $\qquad$ $y = \frac{1}{2}f(x)$

Vertical stretch

> 1

$y = af(x)$

horizontal stretch

< 1

$y = |2x|$

$(1,1)\ (1,2)$

$(-8,8)\ (-8,16)$

$y = \left|\frac{x}{2}\right|$

$(1,1)\ (1,\frac{1}{2})$

$(-8,8)(-8,4)$

$y = 2\left|\frac{x}{3}\right|$

$(1,1)(1,\frac{2}{3})$

$(-8,8)\ (-8,\frac{16}{3})$

$5\frac{1}{3}$

$y = 3\left|\frac{x}{4}\right|$

$(1,1)\ (1,\frac{3}{4})$

$(-8,8)(-8,6)$

$y = 4\left|\frac{\overset{8}{x}}{3}\right|$

$(1,1)\ (1,1\frac{1}{3})$

$(-8,8),(-8,10\frac{2}{3})$

$\frac{4}{1} \times \frac{8}{3} = \frac{32}{3}$

$$y = 2\left|\frac{x}{\frac{1}{4}}\right| \qquad \frac{\frac{3}{4}}{\frac{1}{4}} \qquad \frac{3}{4} \times \frac{4}{1}$$

$$y = 2\left|4x\right|$$

$$y = a\left|\frac{x-h}{b}\right| + k$$

If a > 1, then there is a vertical stretch.

If b > 1, then there is a horizontal stretch.

$$\frac{y-k}{a} = \left|\frac{x-h}{b}\right|$$

$$y - k = a\left|\frac{x-h}{b}\right|$$

$$y = a\left|\frac{x-h}{b}\right| + k$$

Rigid transformations - things don't change shape
translations, reflections

$\{ \text{---}, \text{--}, \} \rightarrow$ 2nd 1

Non-rigid transformations - things do change shape
shrinks, stretches

Scale factor - $\frac{a}{b}$ the amount a function is stretched or shrunk

Ex. B

Time (s) x	Height (m) y
0.54	0.05
0.58	0.18
0.62	0.29
0.66	0.39
0.70	0.46
0.74	0.52
0.78	0.57
0.82	0.59
0.86	0.60

Time (s) x	Height (m) y
0.90	0.59
0.94	0.57
0.98	0.52
1.02	0.46
1.06	0.39
1.10	0.29
1.14	0.18
1.18	0.05

Parent function: $y = x^2$

New vertex: $(\underset{h}{0.86}, \underset{k}{0.60})$

Reflected? $y = -x^2$

New function: $y = -(x - 0.86)^2 + 0.60$

$y = -5(x-0.86)^2 + 0.60$

p. 214 #1–5

Stretch or Shrink of a Function

A **stretch** or a **shrink** is a transformation that expands or compresses a graph either horizontally or vertically.

Given the graph of $y = f(x)$, the graph of

$$\frac{y}{a} = f(x) \quad \text{or} \quad y = af(x)$$

is a vertical stretch or shrink by a factor of a. When $a > 1$, it is a stretch; when $0 < a < 1$, it is a shrink. When $a < 0$, a reflection across the x-axis also occurs.

Given the graph of $y = f(x)$, the graph of

$$y = f\left(\frac{x}{b}\right) \quad \text{or} \quad y = f\left(\frac{1}{b} \cdot x\right)$$

is a horizontal stretch or shrink by a factor of b. When $b > 1$, it is a stretch; when $0 < b < 1$, it is a shrink. When $b < 0$, a reflection across the y-axis also occurs.

$y=|x|$  $y=|x-h|+k$ $y=-f(x)$

$y=x^2$  $y=(x-h)^2+k$ $y=f(-x)$

$y=\sqrt{x}$  $y=\sqrt{(x-h)}+k$

1 a) $y=|x|+2$

b) $y=|x|-5$

c) $y=|x+4|$

d) $y=|x-3|$

e) $y=|x|-1$

f) $y=|x-4|+1$

g) $y=|x+5|-3$

h) $y=|x-6|3$

i) $y=-\left|\frac{x}{4}\right|$

j) $y = (x-5)^2$

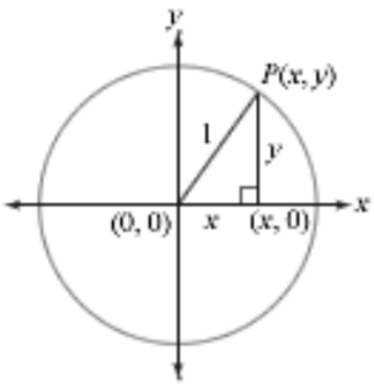
k) $y = -\left|\frac{x+4}{2}\right| - 2$

l) $y = -|x+4| + 3$

m) $y = -(x-3)^2 + 5$

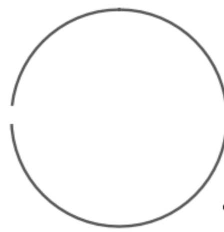
n) $y = \pm\sqrt{(x-4)} + 4$

4.7 Transformations and the circle family p. 217 12/7



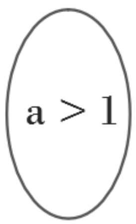
The equation of a unit circle (radius = 1) is

$$x^2 + y^2 = 1 \quad \text{or} \quad y = \pm\sqrt{1-x^2}$$



$$y = +\sqrt{1-x^2}$$

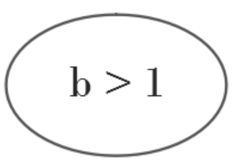
$$y = -\sqrt{1-x^2}$$



An ellipse at the origin

$$\left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

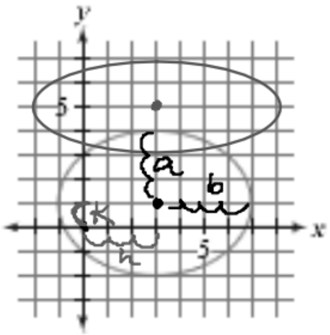
a = vertical dilation (stretch)
b = horizontal dilation



An ellipse translated away from the origin

$$\left(\frac{x-h}{b}\right)^2 + \left(\frac{y-k}{a}\right)^2 = 1 \quad \text{or} \quad y = 1 \pm a\sqrt{1-\left(\frac{x-h}{b}\right)^2} + k$$

Ex. A What is the equation of this ellipse?



$$y = 1 \pm 3\sqrt{1 - \left(\frac{x-3}{4}\right)} + 1$$

$$\left(\frac{x-h}{b}\right)^2 + \left(\frac{y-k}{a}\right)^2 = 1$$

$$\left(\frac{x-3}{4}\right)^2 + \left(\frac{y-1}{3}\right)^2 = 1$$

$$\left(\frac{x-3}{5}\right)^2 + \left(\frac{y-5}{2}\right)^2 = 1$$

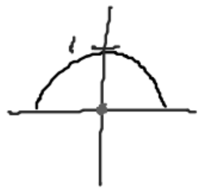
$(3, 5)$
h, k

$b \leftrightarrow 5$

$a \updownarrow 2$

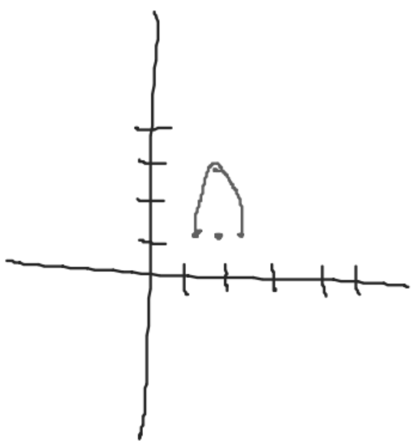
Ex. B $f(x)=\sqrt{1-x^2}$ $\qquad g(x)=2f(3(x-2))+1$

$\cdot\,\frac{3}{1} \div \frac{1}{3}$



$$g(x)=\underset{a}{2}\sqrt{1-[\underset{\frac{1}{b}}{3}(x-\underset{h}{2})]^2}+\underset{k}{1}$$

$$b=\frac{1}{3}$$



$$g(x)=\underset{a}{4}f(\underset{\frac{1}{b}}{2}(x-\underset{h}{1}))+\underset{k}{2}$$

$b=\frac{1}{2}$

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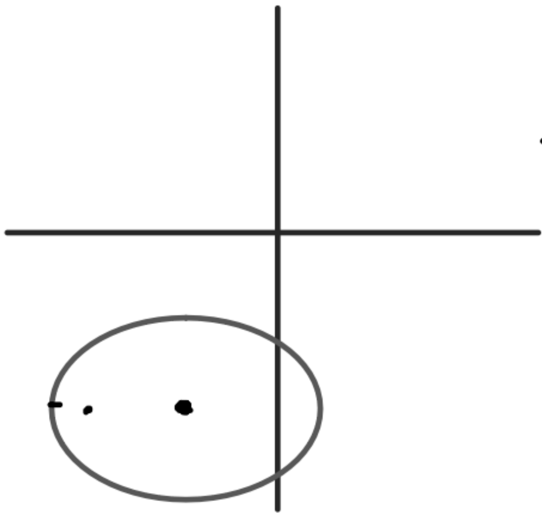

$-2,-4$

h,k

$a=2$

$b=3$

$$\left(\frac{x+2}{3}\right)^2 + \left(\frac{y+4}{2}\right)^2 = 1$$

$$\sqrt{\ }\ \left(\frac{y+4}{2}\right)^2 = \sqrt{\ }\ 1 - \left(\frac{x+2}{3}\right)^2$$

$$\times 2 \quad \frac{y+4}{2} = \sqrt{1-\left(\frac{x+2}{3}\right)^2} \quad \times 2$$

$$y+4 = \pm 2\sqrt{1-\left(\frac{x+2}{3}\right)^2}$$

-4

$$y = \pm 2\sqrt{1-\left(\frac{x+2}{3}\right)^2} - 4$$

$y_1 = 2$

$y_2 = -2$

Transformations of Functions and Relations

Translations

The graph of $y = k + f(x - h)$ translates the graph of $y = f(x)$ h units horizontally and k units vertically.

or

Replacing x with $(x - h)$ translates the graph h units horizontally.
Replacing y with $(y - k)$ translates the graph k units vertically.

Reflections

The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ across the y-axis. The graph of $y = -f(x)$ is a reflection the graph of $y = f(x)$ across the x-axis.

or

Replacing x with $-x$ reflects the graph across the y-axis. Replacing y with $-y$ reflects the graph across the x-axis.

Stretches and Shrinks

The graph of $y = af\left(\frac{x}{b}\right)$ is a stretch or shrink of the graph of $y = f(x)$ by a vertical scale factor of a and by a horizontal scale factor of b.

or

Replacing x with $\frac{x}{b}$ stretches or shrinks the graph by a horizontal scale factor of b.
Replacing y with $\frac{y}{a}$ stretches or shrinks the graph by a vertical scale factor of a.

4.8 Compositions of functions p. 225 12/8

composition - made of parts

$f(x) \quad g(x) \qquad f(g(x)) \quad g(f(x))$

$f(x) = 2x + 7 \qquad g(x) = 3x^2$

$f(g(x)) = 2(3x^2) + 7$

$g(f(x)) = 3(2x+7)^2$

Ex. A $f(x) = \frac{3}{4}x - 3$ $g(x) = |x|$

$I = PRT$

$f(g(x)) = \frac{3}{4}(|x|) - 3$

$g(f(x)) = |(\frac{3}{4}x - 3)|$

$u_n = \left(1 + \frac{0.07}{12}\right)u_{n-1} - 250$

Ex. B $A(x) = \left(1 + \frac{0.07}{12}\right)x - 250$

APR 7% compounded monthly on a loan w/a monthly payment of \$250.

\$14,837.50 12/9

principal = \$15,000

a) $A(15000)$ 1 mo ~~1 yr~~ / 12 mo

u_0

b) $A(A(20000))$ 2 mo $p = \$20{,}000$ \$19,732.56

c) $A(A(A(18000)))$ 3 mo $p = \$18{,}000$ \$17,562.46

d) $A(A(x)) = \left(1+\frac{0.07}{12}\right)\left[\left(1+\frac{0.07}{12}\right)x - 250\right] - 250$

$$= \left(1+\frac{0.07}{12}\right)^2 x - 250\left(1+\frac{0.07}{12}\right) - 250$$

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$\xleftrightarrow{\quad h \quad}$ $\qquad \updownarrow k$